Simultaneous Mass and Heat Transfer in the Flow of Gases Past Single Spheres

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The results obtained by the evaporation of water and nitrobenzene in air from celite spheres in conjunction with the prevailing surface temperature have permitted the simultaneous determination of mass and heat transfer factors. The spheres used were 1.42, 1.88, and 2.00 in. in diameter. The experimental results of this study show that an essentially direct correspondence exists for mass and heat transfer in the flow of fluids past single spheres.

Mass, heat, and momentum transfer in the flow of gases through granular solids point to similarities prevailing for mass and heat transfer. Momentum transfer, as pointed out first by Gamson et al. (*3*), does not show a direct correspondence with mass or heat transfer. This behavior can be explained by the fact that the transfer of momentum to particles is the aggregate contribution of skin friction and form drag. A resolution of the total drag into these contributing components would prove highly desirable for the establishment of mass, heat, and momentum transfer analogies beyond studies concerned with the flow of fluids through circular conduits (*4*).

The existence of simultaneous mass and heat transfer taking place from bluff objects presents a condition that is quite similar from an experimental point of view. Indications that mass and heat transfer are related have been reported in the flow of gases through packed beds (*1, 3, 5*). At this time it would prove highly desirable to verify experimentally the existence of any possible correspondence on single spheres.

Inasmuch as a theoretical approach to this subject for the establishment of local transfer coefficients is not only too complex but also of questionable practical value, experimental results can be used in conjunction with the *j*-factors proposed by Chilton and Colburn (*2*) to represent the transport of mass and heat between the surface of a sphere and a fluid flowing past it.

EXPERIMENTAL EQUIPMENT AND PROCEDURE

Simultaneous mass and heat transfer studies were concerned with the evaporation of water and nitrobenzene from spheres into a stream of air. The spheres were produced from a celite preparation and were ground to exact diameters of 1.42, 1.88, and 2.00 in. Two copper-constantan thermocouples (No. 36 B and S gauge) were permanently attached to each sphere by first drilling two small holes perpendicular to each other through major diameters. The thermocouple wires were then inserted through until the couple junctions reached a point just below the opposite surface whereupon they were fastened in place with a few drops of epoxy resin. The details of the sphere are presented diagrammatically in Figure 1.

Each sphere was supported horizontally with a hollow stainless steel rod, threaded on one end to engage a tapped enlargement of one of the drilled major diameters. This arrangement made possible the rotation of the sphere, thus permitting measurement of the surface temperature distribution. The thermocouples were connected to a precision potentiometer.

The spheres were positioned just above the discharge end of an 8-in. round duct, 4 ft. in length, and provided at the bottom with a calming section. Air was introduced into the calming section from an air blower. The flow of air at the discharge end of the duct was measured by traversing a pitot tube connected to a type MM-2 micromanometer.

The porous spheres were saturated with either water or nitrobenzene, placed in the test section, and periodically weighed in order to determine the rate of evaporation. Dry-and wet-bulb temperatures as well as the barometric pressure measurements were taken in addition to the sphere surface temperatures. Constant rates of evaporation for water lasted beyond 2 hr., while those of nitrobenzene were even longer.

INTERPRETATION OF RESULTS

A number of preliminary runs failed to show a significant trend of temperature variation around the sphere. A random spread of less than 1°F. was noted during the evaporation of water, while no temperature spread was detected when nitrobenzene was used.

The experimental information was analyzed to determine the mass and heat transfer factors proposed by Chilton and Colburn (*2*):

TABLE 1. EXPERIMENTAL DATA AND CALCULATED RESULTS

Run no.	Atm. press., mm.	Temperature, °F. t_d	t_w	t_s	G, lb./hr. sq. ft.	$r \times 10^4$, lb-moles/ hr.	q, B.t.u./ hr.	k_g, lb-moles/ hr. sq. ft. atm.	h, B.t.u./ hr. sq. ft. °F.	j_d	j_h	N_{Re}
	Air-water (d_s = 2.00 in.) ($0.613 \leq N_{Sc} \leq 0.627$) ($0.717 \leq N_{Pr} \leq 0.720$)											
1	749.3	99.0	76.0	76.8	3280	8.25	15.71	1.206	8.140	0.00738	0.00822	11,950
2	753.4	100.1	70.6	71.1	3176	10.65	20.40	1.246	8.061	0.00793	0.00842	11,580
3	760.1	102.0	74.3	75.0	2990	10.01	19.03	1.164	8.101	0.00784	0.00891	10,900
4	745.6	100.4	80.5	80.5	2830	6.73	12.74	1.290	7.310	0.00901	0.00853	10,350
5	745.6	100.5	79.3	79.3	2175	5.95	11.31	0.938	6.131	0.00853	0.00925	7,950
6	745.7	100.5	79.2	79.3	2532	7.06	13.41	1.110	7.275	0.00867	0.00943	9,250
7	748.4	97.5	74.0	73.8	1650	5.94	11.32	0.803	5.484	0.00975	0.01098	6,050
8	748.4	95.5	73.4	73.0	905	4.20	7.64	0.623	4.102	0.01378	0.01501	3,320
9	748.5	95.5	73.4	73.0	1346	5.02	9.57	0.747	4.899	0.01110	0.01201	4,930
10	748.7	99.5	76.0	76.0	1131	4.61	8.79	0.673	4.290	0.01175	0.01249	4,130
11	748.7	99.5	76.0	76.0	1932	5.96	11.39	0.871	5.558	0.00892	0.00949	7,050
	Air-water (d_s = 1.42 in.) ($0.614 \leq N_{Sc} \leq 0.617$) ($0.720 \leq N_{Pr} \leq 0.722$)											
12	752.5	94.2	76.1	75.8	947	1.94	3.68	0.802	4.549	0.01701	0.01602	2,440
13	752.5	92.2	76.0	75.8	568	1.26	2.40	0.522	3.320	0.01831	0.01941	1,478
14	752.5	94.2	76.1	75.8	679	1.42	2.70	0.591	3.341	0.01732	0.01690	1,768
15	752.6	94.2	76.1	75.8	841	1.69	3.21	0.701	3.961	0.01661	0.01562	2,190
16	752.6	96.0	76.5	76.2	2204	3.28	6.24	1.341	7.154	0.01121	0.01078	5,750
17	752.5	95.0	76.9	76.2	1139	2.02	3.86	0.829	4.670	0.01450	0.01374	2,970
18	752.6	96.0	76.5	76.2	1678	2.47	4.73	1.011	5.422	0.01201	0.01275	4,380
	Air-nitrobenzene (d_s = 1.88 in.) ($1.850 \leq N_{Sc} \leq 1.851$) ($0.715 \leq N_{Pr} \leq 0.716$)											
19	752.2	100.4	82.0	99.1	3262	0.395	0.855	0.601	8.561	0.00801	0.00857	11,180
20	752.0	99.9	81.5	98.6	2840	0.354	0.782	0.549	7.820	0.00826	0.00898	9,700
21	752.2	99.3	80.7	98.0	2200	0.298	0.670	0.472	6.702	0.00915	0.00994	7,530
22	752.2	97.8	80.7	96.5	1669	0.252	0.595	0.424	5.950	0.01087	0.01163	5,720
23	752.0	97.8	80.7	96.5	1071	0.188	0.445	0.315	4.459	0.01251	0.01358	3,700
24	752.0	97.9	80.8	96.6	785	0.157	0.370	0.315	3.711	0.01430	0.01542	2,700
25	752.2	97.9	80.8	96.6	579	0.134	0.316	0.224	3.165	0.01651	0.01750	2,010

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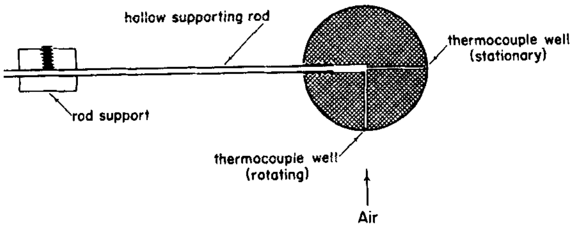

Fig. 1. Diagrammatic sketch of sphere.

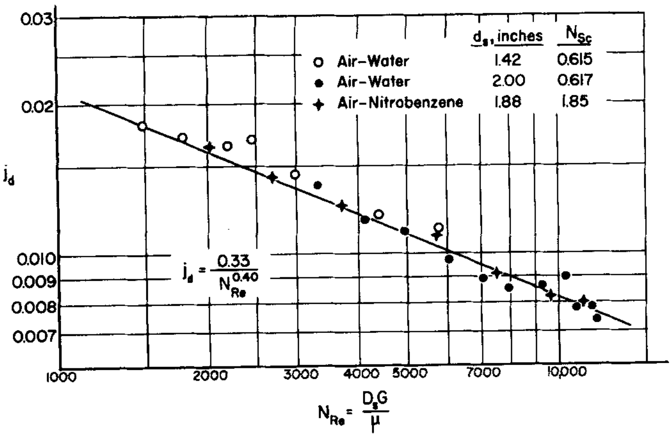

Fig. 3. Relationship of heat transfer factor and modified Reynolds number for single spheres.

$$j_d = \frac{k_g p_{gf}}{G/M}\left(\frac{\mu}{\rho D_v}\right)_f^{2/3} \qquad (1)$$

$$j_h = \frac{h}{c_p G}\left(\frac{c_p \mu}{k}\right)_f^{2/3} \qquad (2)$$

In order to process the nitrobenzene runs it became necessary to utilize the vapor-pressure data reported by Lynch and Wilke (*7*) for the temperature range 43.0° to 73.6°F. To extend this information to the temperatures prevailing in this study their vapor-pressure data, in conjunction with the normal boiling point of nitrobenzene, were used to develop the following vapor-pressure equation:

$$\log p_{mm} = 7.81362 - \frac{2218.4}{T, °\text{K.}} - \frac{87{,}000}{(T, °\text{K.})^2} \qquad (3)$$

In addition the diffusivity for the air-nitrobenzene system reported by Lee and Wilke (*6*) has been used to calculate a Schmidt number, $N_{Sc} = 1.85$. The basic data of this investigation are presented in Table 1.

The mass transfer factors of this study for the air-water and air-nitrobenzene systems are presented in Figure 2, as functions of the modified Reynolds number, N_{Re}. The results for these two systems coincide and produce the single relationship

$$j_d = \frac{0.33}{N_{Re}^{0.40}} \qquad (4)$$

A corresponding treatment of the data has produced for both systems the heat transfer factors presented in Figure 3. These results also produce a unique relationship which can be expressed as

$$j_h = \frac{0.35}{N_{Re}^{0.40}} \qquad (5)$$

From Equations (4) and (5) it follows that the ratio $(j_h)/(j_d) = (0.35)/(0.33) = 1.060$. From the comparable studies concerned with the evaporation of water from packed beds of celite spheres Gamson, Thodos, and Hougen (*3*) have reported a ratio of $j_h/j_d = 1.076$.

Equations (4) and (5) are essentially identical, and consequently it can be concluded that the processes of mass and heat transfer for the flow of air past single spheres bear essentially a direct correspondence to each other.

The mass and heat transfer relationships resulting from Equations (4) and (5) are compared with those reported by others for studies with single spheres. The results of Equation (4) for mass transfer are found to be in good agreement with the j_d-factor relationships reported by Maisel and Sherwood (*8*) and Williams (*10*), while the j_h-factor correlation presented by McAdams (*9*) for single spheres also closely agrees with the relationship resulting from Equation (5).

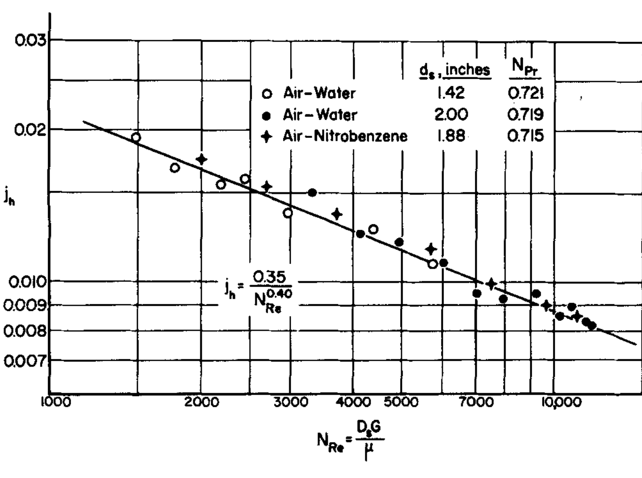

Fig. 2. Relationship of mass transfer factor and modified Reynolds number for single spheres.

NOTATION

c_p	=	heat capacity, B.t.u./lb. °F.
d_s	=	sphere diameter, in.
D_s	=	sphere diameter, ft.
D_v	=	diffusion coefficient of transferable component, sq.ft./hr.
G	=	mass velocity, lb./hr. sq. ft.
h	=	heat transfer coefficient, B.t.u./hr. sq. ft. °F.
j_d	=	mass transfer factor, dimensionless
j_h	=	heat transfer factor, dimensionless
k	=	thermal conductivity, B.t.u./hr. ft. °F.
k_g	=	mass transfer coefficient, lb.-moles/hr. sq. ft. atm.
M	=	molecular weight
p_{gf}	=	partial pressure of nontransferable component, atm.
p_{mm}	=	vapor pressure, mm. of mercury
N_{Pr}	=	Prandtl number, $c_p\mu/k$
q	=	rate of heat transfer, B.t.u./hr.
r	=	rate of mass transfer, lb.-moles/hr.
N_{Re}	=	Reynolds number, D_sG/μ
N_{Sc}	=	Schmidt number, $\mu/\rho D_v$
T	=	absolute temperature, °K.

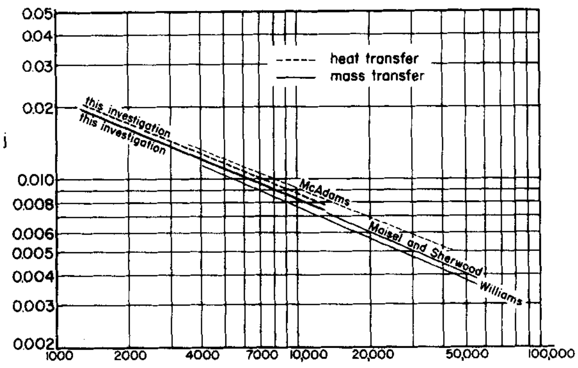

Fig. 4. Comparison of j-factor for single spheres resulting from this study with other available data.

t_d = dry-bulb temperature, °F.
t_s = surface temperature, °F.
t_w = wet-bulb temperature, °F.
μ = viscosity, lb./hr. ft.
ρ = density, lb./cu. ft.

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Size Distribution of Droplets from Centrifugal Spray Nozzles

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Methods for expressing, measuring, and correlating drop-size distribution data for centrifugal spray nozzles are discussed.

A method for collecting spray droplets in liquid nitrogen is described which is rapid and efficient for most sprayed liquids which freeze above —20°C. Comprehensive correlations for drop-size distributions are reported based on 114 runs performed with the liquid nitogen method.

Studies of the kinetics of evaporation or combustion of droplets issuing from a spray nozzle require data on the size distribution of the droplets. Unfortunately these data are not easily obtained because the droplets are very minute and difficult to sample; also it is difficult to express and correlate drop-size data because the drops issuing from a spray nozzle are nonuniform in size.

The purpose of this paper is to report on methods of measuring, expressing and correlating drop-size data which the writers found to be successful in a recent study of grooved-core centrifugal spray nozzles.

DISTRIBUTION FUNCTIONS

A theoretical distribution function for expressing drop-size distributions has not been derived as yet. The best that can be done with experimental data is to express the distribution graphically or to specify a distribution function and the values of the parameters in the distribution equation which will most closely represent the data.

Several of the distribution functions which have been proposed are based on the normal distribution, and only this type will be discussed:

$$f_v(y) = \frac{dv}{Vdy} = \frac{1}{s\sqrt{2\pi}} e^{-\frac{(y-\bar{y})^2}{2s^2}} \tag{1}$$

Equation (1) is the normal distribution equation expressed in terms of the volume of droplets.

The size characteristic is a function of the drop diameter. However if the drop diameter itself is chosen as the size characteristic, experimental data will not fit the distribution. Three size characteristics which have been successfully used are the log of the drop diameter (*6*), the upper limit characteristic (*10*), and the square-root of the drop diameter (*15*).

The log-normal distribution and the square-root normal distribution have two parameters, a mean and a standard deviation, which can be adjusted to fit a particular set of data. The upper-limit distribution has a third parameter, the maximum stable drop size, which permits more flexibility in fitting experimental data.

From a theoretical standpoint the upper-limit distribution has another advantage over the log-normal distribution and the square-root normal distribution in that it places reasonable limits on the minimum and maximum drop size. However despite the fact that the extreme ends of the distribu-

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